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Question 1:

a). Negation: $a < b$ and $a^2 > b^2$.

Counterexample: $a = -2$, $b = 1$ ($a < b$) and $4 > 1$ ($a^2 > b^2$).

b). Negation: n is odd and $(n-1)/2$ is even.

Counterexample: $n = 5$ (n is odd) and $((5-1)/2)$ is 2 which is even.

Question 2:

a) Counterexample: $m = 2$, $n = 1$. $2(2) + 1$ equates to 5, which is odd but then m is not odd so therefor the statement cannot be correct.

b) Counterexample: $p = 2$ (2 is a prime number), $2^2 - 1$ is odd, which means that the statement cannot be correct.

c) Counterexample: $n = 8$ (8 is even), $8^2 + 1$ is 65 which is not a prime number therefor the statement cannot be correct.

Question 3:

a) The product of all rational numbers must also be a rational number, because any integer is a rational number, and rational number can be represented as fractions, which is an integer dividing an integer. When multiplying fractions, you multiply the two denominators and numerators which are both integers, and the product of two integers is always rational.

b) The quotient of all rational numbers being rational is the same reason as the reason that the product of two rational numbers is also rational, because to divide two fractions you must simply take the reciprocal of the second rational number and then multiply, which does not change the kind of number it is.

c) To subtract a fraction the equation is $(a/b) - (c / d) = (ad - bc) / bd$. In this operation there is only the products of integers and the subtraction of integers, which always produce integers which are rational numbers

d) The sum of two fractions is a rational number, for the same reason that the difference is also a rational number. The summation or subtraction of two integers always produces an integer. So with the original $r + s$ numerator proven to be rational, the division of 2 cannot make the number irrational given that 2 can be represented as $(2/1)$ which is rational, and the division of rational numbers must produce a rational number. Therefor the original equation must produce a rational number.

Question 4:

a) For any integer a, the equation for its consecutive 2 numbers would be $(a+1)$ and $(a+3)$ so the equation $(a) + (a+1) + (a+2)$ must be divisible by three. This can be represented as $(3a+3)$. You can then take the 3 out of the equation and get $3(a+1)$. By its very definition three consecutive numbers must be divisible by 3.

b) The product of any two even integers can be thought of like 2a * 2b, which can be simplified to 4(ab), which is by its nature divisible by 4.

c) If a divides b, there is a number x, that ax = b. If a divides c, then there is a number y that ay=c. If you substitute these into (2b-3c) you obtain (2ax-3ay) which can be simplified to a(2x-3y) This by its nature can be divided by a.

d) If ab divides c then there is a value x that (ab)(x) = c. Rearranged this is b(ax) =c. When substituting this into b divides c you obtain b divides b(ax), which by its nature must be true

e) Counterexample: a = 4, b = 2, c = 6. 4 can divide (2+6), but it cannot divide 2 or 6 individually.

Question 5.

a) Contradiction: For all odd integers a and b, $b^2 – a^2 = 4$.
This is not possible because the lowest distance between two squares of odd numbers is 8, when a=1 and b=3. The gap between any two odd integers will only widen and cannot become smaller, unless the two numbers are the same, then it will be 0. Therefore, the statement is true because the difference cannot be 4.

b) Contradiction: For all prime numbers, a, b, c, $a^2+b^2=c^2$
First take the square root of both sides. We can now say a = sqrt ((b - c) * (b + c)). The only way that sqrt ((b - c) * (b + c)) can be an integer is if (b - c) = (b + c) or (b - c)=1 or (b + c) = 1. For (b - c) to equal (b + c) is not possible unless b and c are both zero, which in that case, they would not be prime.

c) Contradiction: For any rational number, a, b, where b is not 0, and any irrational number r, a + (b * r) is rational
Any number multiplied by an irrational number cannot be rational. This is because if we were to represent b as x/y where y is not zero, and you cannot multiply it with x because you would need to represent x as a fraction.

d) Contradiction: For any integer n, n^2-2 is divisible by 4.
Let 4x = n^2-2, then 4x + 2 = n^2. If n is an odd number this equation is impossible, since 4x+2 must produce an even number. If the number is even, you can say $(2y)^2 = n^2$ and $4x + 2 = 4y^2$. Then divide by two $2x + 1 = 2y^2$. If n is even then it must equal 2x+1 which is odd, which is a contradiction. It is there not possible in either case and is a contradiction

e) Contradiction: For every irrational number, the negative is rational
Since the negative of the number is rational you can rewrite the number such that a/b where b does not equal 0. You may then multiply the equation by -1 and making the original number rational. This cannot happen as there is not way to turn an irrational number rational.
Contraposition: If a number is real, it's negative counterpart is real
The number is real so it can be written as a/b where b is not 0, and then can be multiplied by -1. The number is still real, and therefor the contraposition is true.

f) Contradiction: For every integer n, n^2 is odd, n is even
Since n is odd, n^2 can be written as (2x+1) * (2x+1), which can be simplified to $4x^2+4x+1$. This means that any number has to be odd, because the first two sections of the equation would multiply by 4, making the answer even first, and at the very end there is an add 1 operation which would have to make it odd.
Contraposition: If n is even, then n^2 is even
Since n is even, n^2 can be written as (2x) * (2x), which can be simplified to 4x. Anything multiplied by four must be positive, since anything multiplied by 2 is positive, and 4 is a multiple of two. So, any even number squared must be positive.